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Deflection of "Indeterminate" Structures: Super-Position Method

Main Ideas

•The "indeterminate" structure can be decomposed into "series" (or the sums) of "determinate" structures

• Identifying the primary unknowns (usually are Forces and/or Moments at the supports of the "original, indeterminate" structure)

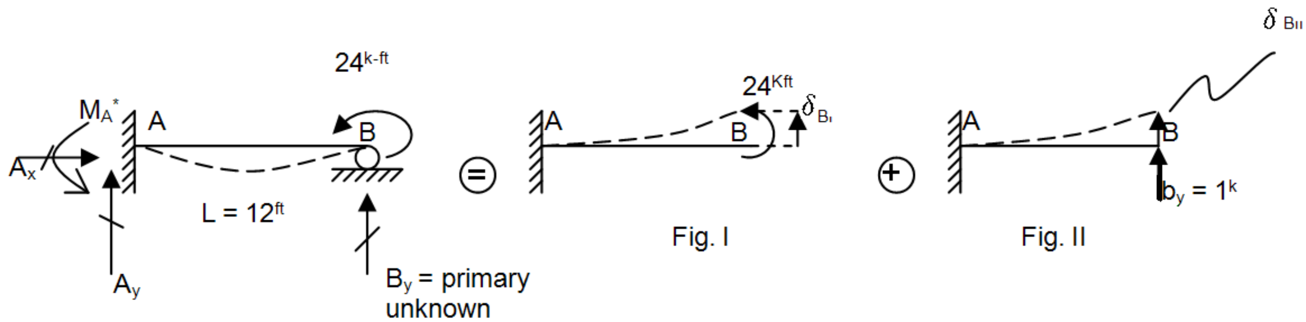
• Establishing/obtaining the corresponding number of equations (usually relate to the displacements and/or rotations at the supports of the "original, indeterminate" structure)

• Solving for the primary unknowns

• Solving for the remaining unknowns (of the "original" structure) by applying static Equilibrium equations

Example 1: Indeterminate Beam

Find all unknown support reactions of the following indeterminate (1^0) beam.



Vertical deflection at point B due to actual load B_y applied at point B

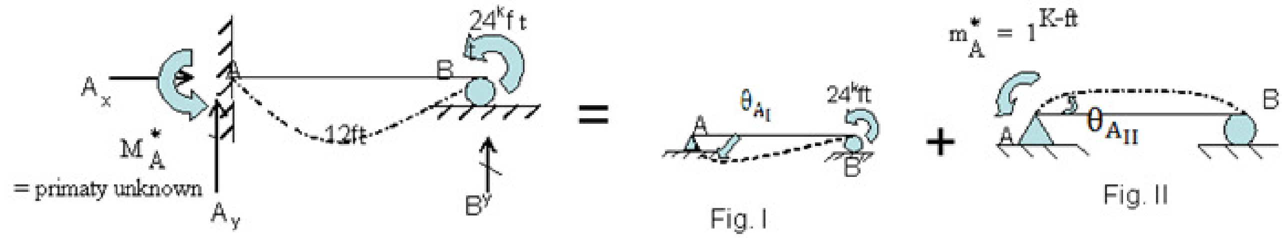
$$(\delta_B)_{\text{Original Structure}} = 0 = \delta_{B_I} + \delta_{B_{II}} * B_y \quad \text{.......................} \quad (81)$$

Vertical deflection at point B (in Fig. II) due to unit load by applied at point B

Notes:

•Since δ_{B_I} (see Fig. I) and $\delta_{B_{II}}$ (see Fig. II) represents the vertical displacement at joint B of the "determinate" structure, they can be easily computed by using V.W. method (see Examples #8, #9 of V.W.)
•After that, B_y can be solved from Eq. (81)
•Finally, the remaining unknown reactions at support A (of original structure) can be found from Static Equilibrium.

Alternate Solution for Example 1



$$(\theta_A)_{\text{Original Structure}} = 0 = \theta_{A_I} + \theta_{A_{II}} * M_A^* \text{ ....(82)}$$

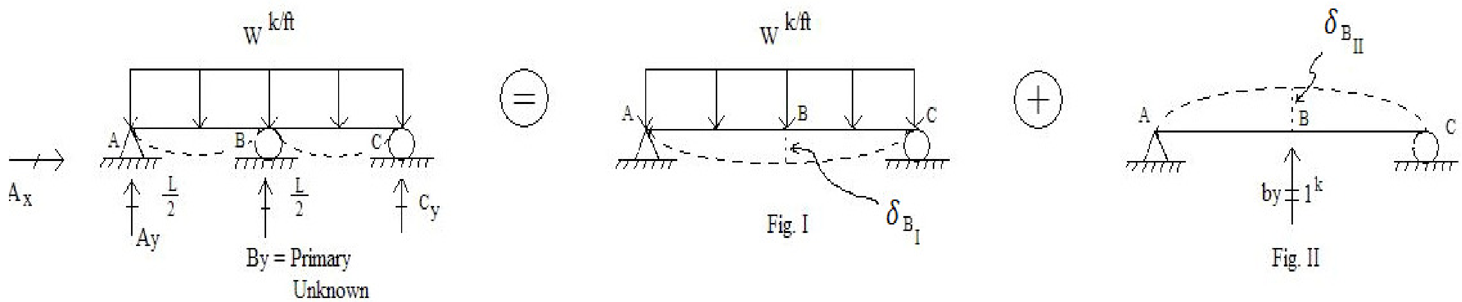
Rotation at joint A due to a UNIT moment m_A^* applied at joint A

Rotation at joint A (in Fig. II) due to actual moment M_A^* applied at joint A

Notes :

- Since θ_{A_I} (see fig. I) and $\theta_{A_{II}}$ (see fig. II) represents the rotation at joint A of the determinate structure they can be easily computed by using V.W. method (see Examples #10, #11 of VW)
- After that the primary unknown M_A^* (moment reaction at support A of the original structure) can be computed from Eq. (82)
- Finally the remaining unknown support reactions (of original structure) can be found by using Static Equilibrium equations.

Example 2: Indeterminate Beam



Find the unknown support reactions at support joints A, B and C of the "original" structure

$$(\delta_B)_{\text{Original Structure}} = 0 = \delta_{B_I} + \delta_{B_{II}} * B_y \quad \text{.......................................} (83)$$

Notes:

- Since δ_{B_I} (see Fig. I) and $\delta_{B_{II}}$ (see Fig. II) represents the vertical displacement at joint B of the determinate beam, they can be easily computed by using V.W. method (see Examples #5, #12 of V.W.)

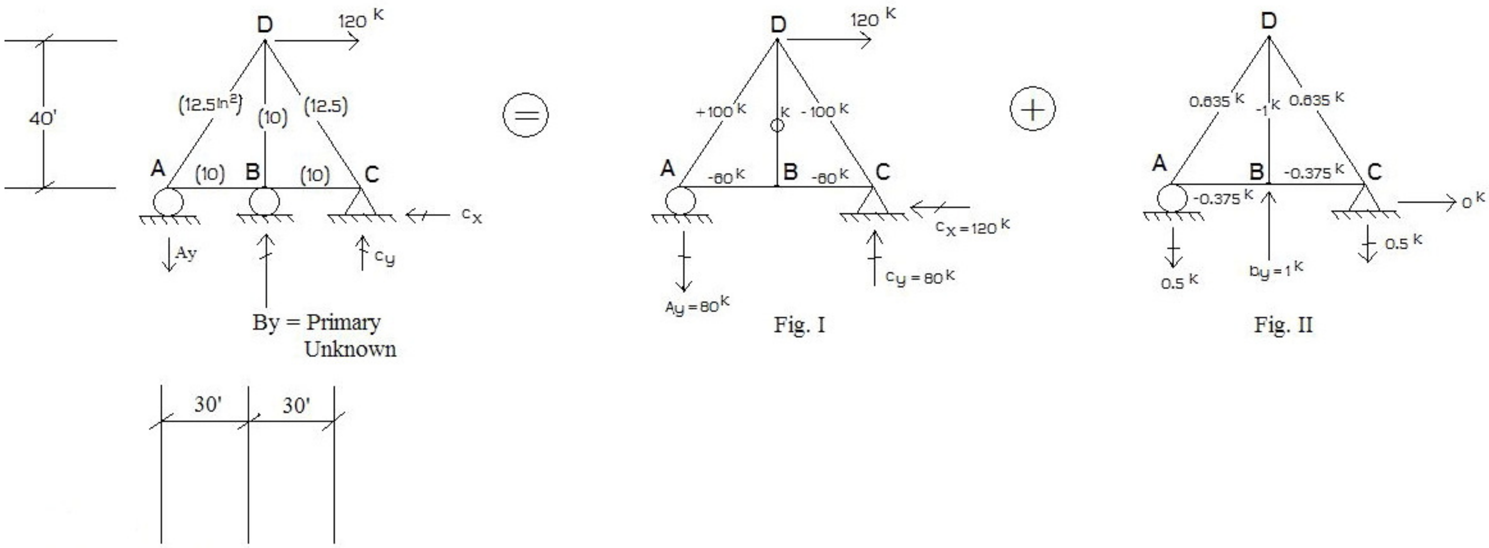
- Then, the <u>"primary"</u> unknown reaction By can computed from Eq. (83)

- Finally, all remaining unknown reactions at supports A, B, C (of the original structure) can be found from static equilibrium equations.

Example 3: Indeterminate (1^o) Truss

Compute all reactions & bar forces for the truss (shown in the figure).

Given: $E = 30{,}000 \text{ k/in}^2$
Cross-sectional areas (in in^2) are shown in parenthesis



$$(\delta_B)_{\text{Original Structure}} = 0 = (\delta_{Bv})_I + (\delta_{Bv})_{II} * B_y \qquad (84)$$

Where: Obtain from Fig. II, from Fig. I

$$(Q = 1^K \uparrow)(\delta_{Bv})_I = \sum_{i=1}^{\text{All members}} F_{Q_i} * \left(\frac{F_p * L}{AE}\right)_i = \frac{+135}{E}\frac{K^2 \text{ ft}}{in^2}$$

$$(Q = 1^K \uparrow)(\delta_{Bv})_{II} = \sum_i F_{Q_i} * \left(\frac{F_Q * L}{AE}\right)_i = \frac{+7.85}{E}\frac{K^2 \text{ ft}}{in^2} \qquad (85)$$

Substituting Eq. (85) into Eq. (84), one obtains: $B_y = -16.93^K$ (downward)

Example 4: Indeterminate (1°) Frame (Similar to Example 7, on V.W.)

Given: $E = 30{,}000\ ^{k}/_{in^2}$; $I = 288\ in^4$

Find : All unknown support reactions.

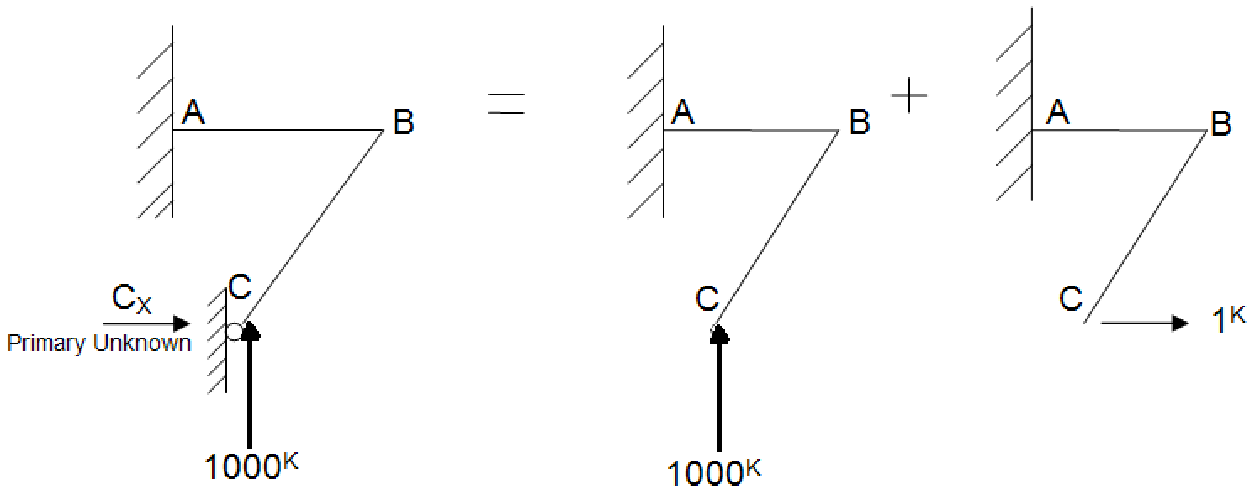


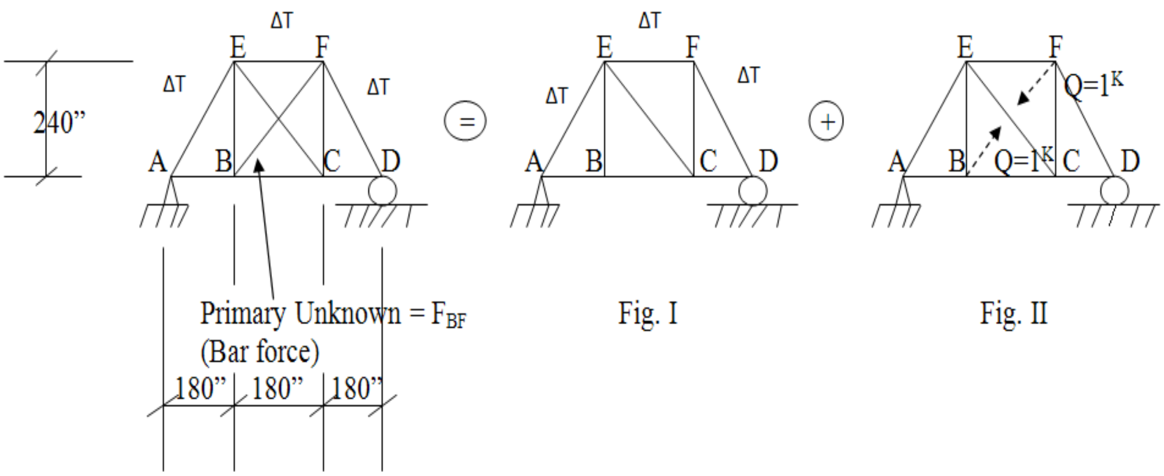
Fig. I

Fig. II

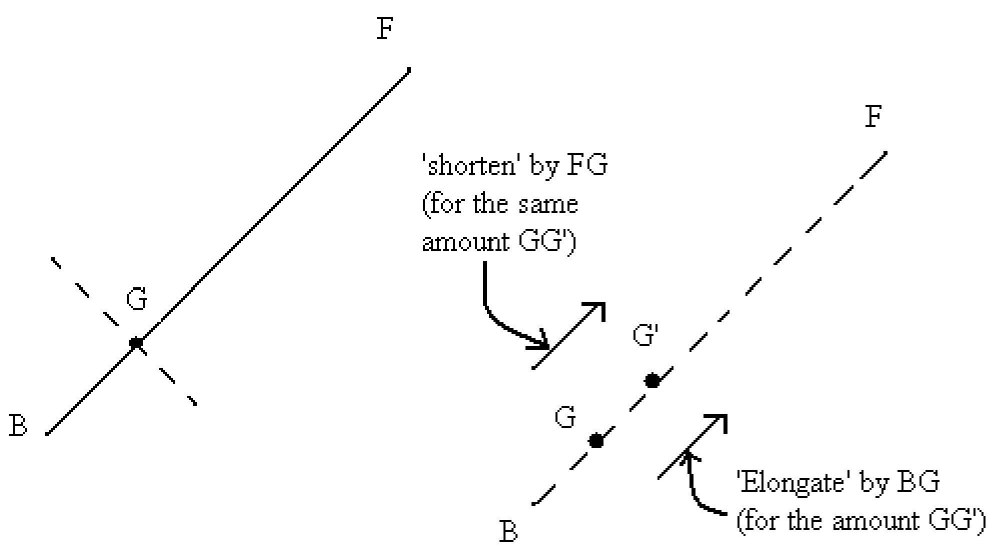
$$(\delta_{C_{Horiz}})_{\text{original Structure}} = 0 = \delta_{C_I} + \delta_{C_{II}} * C_x \ \ldots\ldots\ldots\ldots\ldots\ldots \quad (86)$$

Example 5: Indeterminate (1°) Truss (due to extra member)

Given: $\Delta T = +60°F$ for bars AE, EF, and FD; $\alpha = \frac{1}{150,000}$ per °F; $E = 30,000\ {}^{K}/_{in^2}$
Cross - sectional areas (in^2) are given in parentheses = 10 in^2 (for each bar).



$$(\delta_{BF})_{\text{Original Structure}} = \left(\begin{matrix}\text{Relative displacement}\\ \text{between joint B and F}\end{matrix}\right)_{\text{Orig.structure}} = 0 = (\delta_{BF})_{I} + (\delta_{BF})_{II} * F_{BF} \quad (87)$$



Example 6: Indeterminate (2^0) of Frame structure

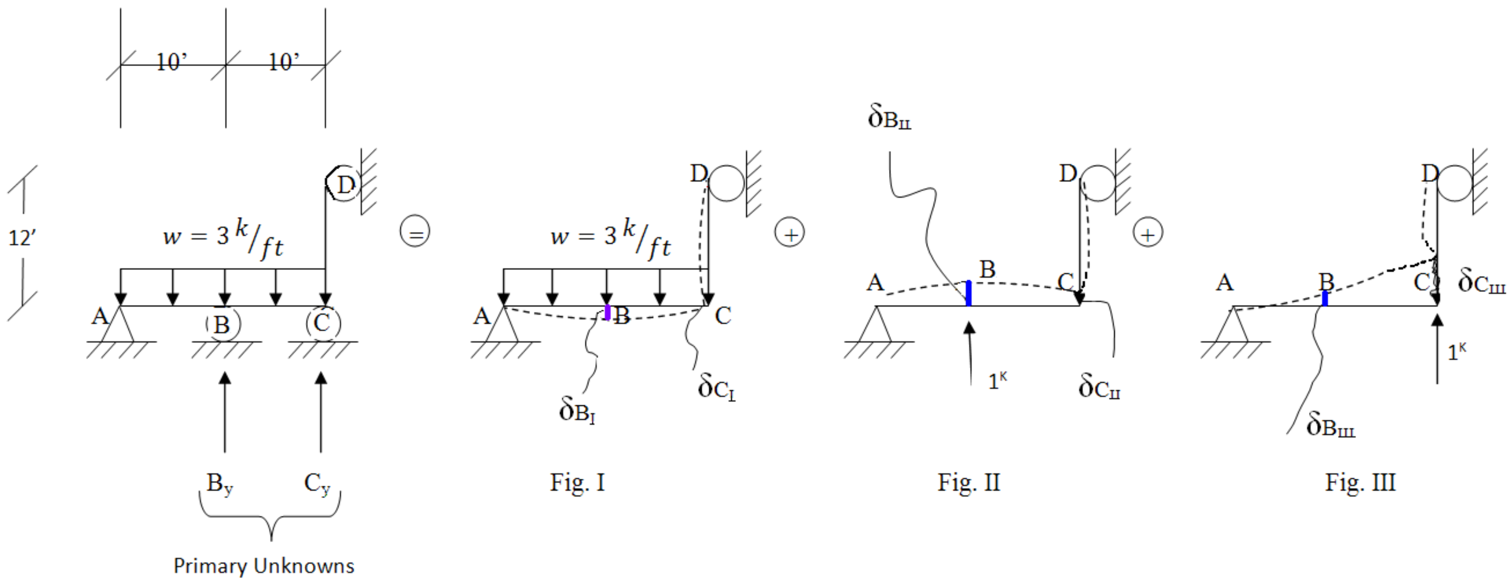


Fig. I Fig. II Fig. III

$$(\delta_B)_{\text{Orig. Structure}} = 0 = \delta_{B_I} + \delta_{B_{II}} * B_y + \delta_{B_{III}} * C_y \qquad (88^A)$$

$$(\delta_C)_{\text{Orig. Structure}} = 0 = \delta_{C_I} + \delta_{C_{II}} * B_y + \delta_{C_{III}} * C_y \qquad (88^B)$$

Fixed End Moments (FEM) Due to Applied Loads

Using superposition principle, one obtains:

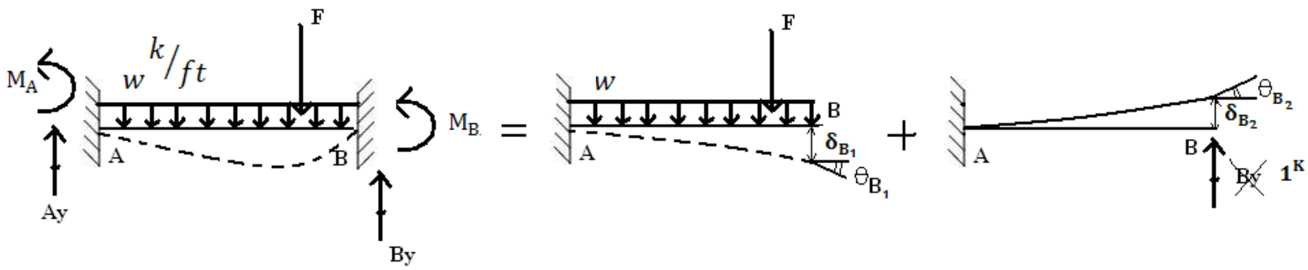


Figure 0 Figure 1 Figure 2

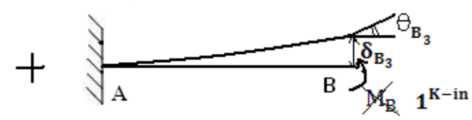


Figure 3

$$(\delta_{B_{vertical}})_o \equiv (\delta_{B_{vertical}})_1 + (\delta_{B_{vertical}})_2 * B_y + (\delta_{B_{vertical}})_3 * M_B$$

$$(\theta_B)_o \equiv 0 = (\theta_B)_1 + (\theta_B)_2 * B_y + (\theta_B)_3 * M_B$$

From the above 2'key' equations, one can solve for the unknown reactions B_y and M_B (At the final support B). Then, from statics equilibrium equations, the unknown support reactions A_y and M_A (At the final support A) can also be found.

FEM (say, earthquake) Support Vertical Displacement

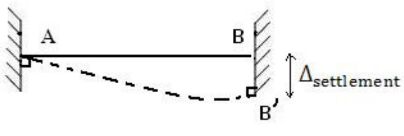


Figure 0

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Figure 1

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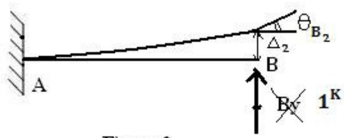

Figure 2

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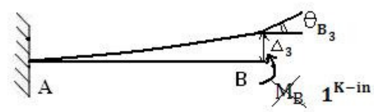

Figure 3

$$\Delta_{B_0} \equiv \Delta_{settlement} = \Delta_{B_2} * B_y + \Delta_{B_3} * M_B \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .(1)$$

$$\theta_{B_0} \equiv 0 = \theta_{B_2} * B_y + \theta_{B_3} * M_B \ldots\ldots\ldots\ldots\ldots\ldots\ldots .\ldots\ldots\ldots .(2)$$

Where using Moment Area Theorems, or

$$\left.\begin{aligned} \Delta_{B_2} &= \frac{L^3}{3}; \qquad \theta_{B_2} = \frac{L^2}{2} \\ \Delta_{B_3} &= \frac{L^2}{2}; \qquad \theta_{B_3} = L \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Using Eq.(3), then Eqs.(1-2) can be simultaneously solved for

$$\boxed{B_y = \frac{+12\Delta_{settlement}}{L^3}}$$

$$\boxed{M_B = \frac{-6\Delta_{settlement}}{L^2}}$$

FEM (say, earthquake) Support Rotation

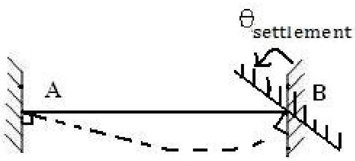

Figure 0

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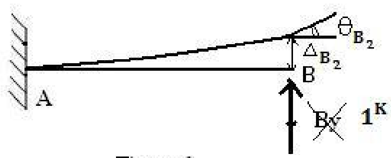

Figure 2

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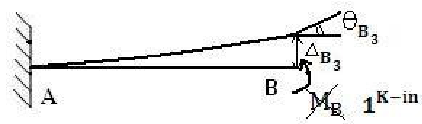

Figure 3

$$\Delta_{B_0} \equiv 0 = \Delta_{B_2} * B_y + \Delta_{B_3} * M_B \quad (1)$$

$$\theta_{B_0} \equiv \theta_{settlement} = \theta_{B_2} * B_y + \theta_{B_3} * M_B \quad (2)$$

Hence:

$$\boxed{M_B = \frac{4\theta_{settlement}}{L}}$$

$$\boxed{B_y = \frac{-6\theta_{settlement}}{L^2}}$$